

## PROBLEM 6

**Atlantis III: Twin Rivers****Input file:** twinin.txt**Output file:** twinout.txt**Time and memory limits:** 1.5 seconds, 1 GB

After years of regulatory delays, King Triton has finally established the city of New Atlantis. To honour the old city, the citizens have chosen a location that features not one but two rivers running through the city.

The city can be modelled as 3 horizontal segments representing the strips of land that the residents live in, labelled Strip 1, 2 and 3. The strips of land are separated by rivers that are each 1 km wide, labelled River 1 and 2. The city is  $L$  km long from left to right and King Triton has built  $N$  bridges, each spanning one of the rivers. The  $i$ th bridge spans river  $R_i$  at a point  $B_i$  km from the left of the city. To prevent issues in the waterways, residents of New Atlantis are not permitted to swim and can only cross at bridges.

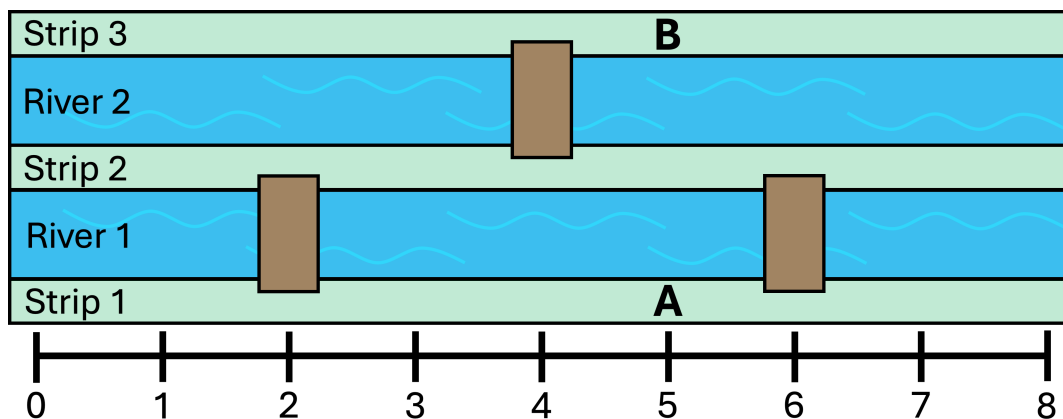


Figure 1: A possible city layout with 3 bridges. This layout corresponds to Sample Input 1.

Residents regularly travel between locations and always take the shortest route, travelling only on land or across bridges. King Triton has identified  $T$  trips that residents regularly take. The  $j$ th trip starts on Strip 1, at a point  $X_j$  km from the left of the city. The trip finishes on Strip  $S_j$  (either 2 or 3), also at a point  $X_j$  km from the left of the city.

For example in Figure 1, a trip from location A to B corresponds to a trip with  $X_j = 5$  and  $S_j = 3$ . The shortest route for this trip is to walk 1 km right, then cross the 1 km-long bridge across River 1, then walk 2 km left, then cross the 1 km-long bridge across River 2, and finally walk 1 km right. This is a total of 6 km. The strips of land are very thin, and so the vertical distance walked within the strips is ignored. We only count the horizontal distance walked along the strips and the vertical distance walked when crossing bridges.

King Triton wishes to build one new bridge. The bridge must span only one of the rivers and must be built an integer number of kilometres from the left of the city, just like the existing bridges. After building the new bridge, it must be possible to complete every trip. Additionally, to save time for his citizens, he wishes to pick a location so that the sum of the distances of all trips will be as small as possible.

As King Triton’s top advisor, you have been tasked with figuring out what this value is. What is the smallest possible sum of trip distances you can achieve after building one more bridge?

## Subtasks and constraints

Your program will be graded using many secret tests. Every test follows some rules:

- $1 \leq N \leq 200\,000$ .
- $1 \leq L \leq 1\,000\,000$ .
- $1 \leq T \leq 200\,000$ .
- $0 \leq B_i \leq L$  for all  $i$ .
- $1 \leq R_i \leq 2$  for all  $i$ .
- $0 \leq X_j \leq L$  for all  $j$ .
- $2 \leq S_j \leq 3$  for all  $j$ .
- $B_i \leq B_{i+1}$  for all  $i$ . That is, the bridges are given in increasing order of  $B_i$ .
- $X_j \leq X_{j+1}$  for all  $j$ . That is, the trips are given in increasing order of  $X_j$ .
- There is at least one bridge crossing the 1st river.
- No two bridges are the same. That is,  $B_i \neq B_k$  or  $R_i \neq R_k$  for all  $i \neq k$ .

The secret tests are divided into subtasks. Your program must correctly solve **every test** within a subtask to earn the marks for that subtask:

- For Subtask 1 (15 marks),  $N, T, L \leq 2000$ ,  $R_i = 1$  for all  $i$  and  $S_j = 2$  for all  $j$ . That is, all existing bridges cross the 1st river and every trip finishes on Strip 2.
- For Subtask 2 (35 marks),  $R_i = 1$  for all  $i$  and  $S_j = 2$  for all  $j$ . That is, all existing bridges cross the 1st river and every trip finishes on Strip 2.
- For Subtask 3 (30 marks),  $R_i = 1$  for all  $i$  and  $S_j = 3$  for all  $j$ . That is, all existing bridges cross the 1st river and every trip finishes on Strip 3.
- For Subtask 4 (20 marks), no special rules apply.

## Input

Your program must read input from the file `twinin.txt`. When testing on your own computer, this file must be placed in the same folder as your program. We strongly recommend using the solution templates (which you can find on the *Templates & Downloads* page of the competition website) to help you with input and output.

The file `twinin.txt` follows a specific format:

- The 1st line of input contains the integers  $N$  and  $L$ .
- The next  $N$  lines describe the locations of the existing bridges. The  $i$ th of these lines contains the two integers  $B_i$  and  $R_i$ .
- The next line contains the integer  $T$ .
- The next  $T$  lines describe the trips. The  $j$ th of these contains the two integers  $X_j$  and  $S_j$ .

## Output

Your program must write a single integer to the file `twinout.txt`: the smallest possible sum of trip distances (in km) after building one additional bridge.

*Note: For students using C, C++ or Java, please note that the answer may exceed the maximum value that can be stored in an `int` integer type. As such, you should consider using the `long long` integer type in C or C++, or the `long` integer type in Java. Please refer to the solution templates (which you can find on the *Templates & Downloads* page of the competition website) for more details. Python users do not need to consider this.*

**Sample input 1**

3 8  
 2 1  
 4 2  
 6 1  
 4  
 3 3  
 4 3  
 5 3  
 5 2

**Sample input 2**

2 6  
 1 1  
 5 1  
 4  
 0 2  
 3 2  
 3 2  
 6 2

**Sample input 3**

2 4  
 0 1  
 3 1  
 2  
 1 3  
 4 3

**Sample output 1**

13

**Sample output 2**

8

**Sample output 3**

10

**Explanation**

- The 1st sample case is shown in Figure 1. The smallest sum of trip distances can be achieved by building a bridge across River 1 at the point 4 km from the left end of the city:
  - The 1st trip ( $X_1 = 3, S_1 = 3$ ) reduces from 6 km to 4 km.
  - The 2nd trip ( $X_2 = 4, S_2 = 3$ ) reduces from 6 km to 2 km.
  - The 3rd trip ( $X_3 = 5, S_3 = 3$ ) reduces from 6 km to 4 km.
  - The 4th trip ( $X_4 = 5, S_4 = 2$ ) remains 3 km long.
- In the 2nd sample case, every trip finishes on Strip 2. Initially, the trip distances are 3 km, 5 km, 5 km, and 3 km respectively for a total of 16 km. Building a bridge across River 1 at the point 3 km from the left of the city would reduce the sum of distances to 8 km, which is the smallest possible.
- In the 3rd sample case, it is not originally possible to complete the trips because there is no bridge crossing River 2. One option is to build a bridge crossing River 2 at the point 3 km from the left of the city. The trip distances would be 6 km and 4 km, giving a sum of 10 km which is the smallest possible.